Modelling the Magnetic Permeability of General Steels

Magnetic permeability is a critical parameter for the optimisation of induction heating processes. It affects the selection of the operating frequency of the coil required to achieve an ideal current penetration depth, with important consequences for the efficiency and lifetime of the heating system.[1] Available material data in the literature is mostly limited to a single room-temperature value, usually the maximum permeability. However, for ferromagnetic steels the temperature and magnetic field dependencies of the permeability are markedly nonlinear,[1,2] which justifies the need for better predictive modelling. This report provides an overview of the model used in JMatPro® to calculate the magnetic permeability of general steels. Note that CGS units are used throughout the document.

Maximum Permeability as a Function of Temperature

The magnetic permeability, $\mu(H,T)$, describes the response of a material to an external magnetic field[2] as a function of field strength, H, and temperature, T. The key assumption to model this property for steels is to treat ferrite as the only ferromagnetic component in the alloy's microstructure. Its temperature dependence may then be split into contributions arising from changes in the volume fraction of ferrite, $v_f(T)$, and from the intrinsic variation for that phase, $\mu_f(H,T)$:

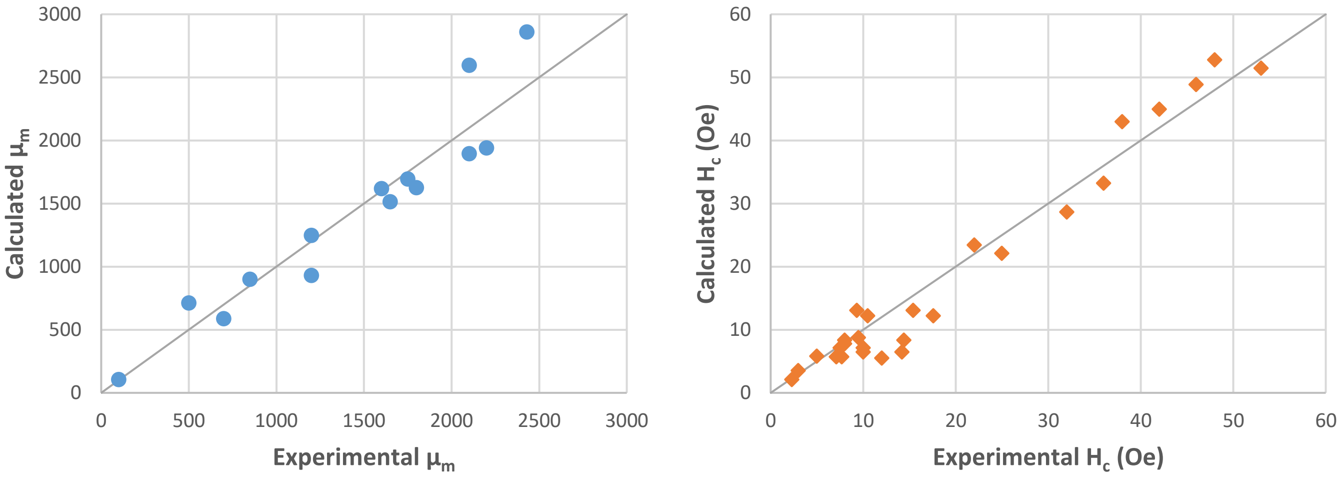
$$\mu(H,T) = v_f(T) \cdot \mu_f(H,T) + 1 - v_f(T) \tag{1}$$

A more practical and commonly used measure is the maximum permeability, $\mu_m(T)$, which lies at different field strength values for different temperatures, but is still assumed to follow the simple linear mixing law of Eq. (1). The term $v_f(T)$ may be determined upon heating from a reaustinitisation procedure based on the knowledge of alloy composition, microstructure, and heating rate. This has been described elsewhere[3] and in the following the focus is on the calculation of the temperature-dependent maximum permeability for ferrite, $\mu_m^f(T)$.

It is convenient to relate the maximum permeability to other more readily available magnetic properties. At room temperature, an empirical relationship may be established in terms of the saturation magnetic flux density, B_s, and the magnetic coercivity, H_c:

$$\mu_m^f = 1 + \frac{B_s}{4H_c} \tag{2}$$

As shown in Fig. 1(a), the above expression provides a very good fit to available room-temperature data for low-alloy steels,[2,4] for which $v_f = 1$. Data at higher temperatures is not as widely available, but it is



(a) *(b)*

Figure 1: *Comparison of predictions for maximum permeability (a) and coercivity (b) with room-temperature experimental measurements. The data used are from Refs. 2, 4, 7, and 8.*

assumed that Eq. (2) is still valid after including the effect of temperature on B_s and H_c. Above the Curie temperature, T_c, the alloy becomes paramagnetic and, accordingly, $B_s = 0$ and μ falls to 1 asymptotically.[2] Below T_c, an approximation to the Brillouin-Langevin equation may be used to describe the temperature variation of the saturation flux density:[5]

$$B_s(T) = B_s^0\left[1-\left(\frac{T}{T_c}\right)^6\right] \tag{3}$$

The value at absolute zero, B_s^0, may also be calculated from the Curie temperature.[6] This approach has the advantage of requiring only the value of T_c, for which the effect of common alloying elements is known.[2] Good results are obtained for low-alloy steels.

An empirical description is used for the temperature dependence of the magnetic coercivity:[7]

$$H_c(T) = H_c^a + H_c^b\left[1-\left(\frac{T}{T_c}\right)^3\right] \tag{4}$$

where H_c^a is a residual term required to make sure that the maximum permeability falls to 1 as the temperature approaches T_c (a good estimate is around 0.1 Oe[2]) and H_c^b is directly proportional to the room-temperature Vickers hardness.[8] This readily available mechanical property effectively integrates different mechanisms contributing to the coercivity. As can be seen in Fig. 1(b), predicted H_c values are in very good agreement with room-temperature measurements for low-alloy steels.[2,7,8]

Combining Eqs. (1) through (4), yields the final expression:

$$\mu_m(T) = 1 + v_f(T)\cdot\frac{B_s^0(1-\tau^6)}{4\left(H_c^a+H_c^b(1-\tau^3)\right)} \tag{5}$$

where $\tau = T/T_c$. The quality of the model is assessed in Fig. 2, which compares calculated μ_m as a function of temperature with experimental results for steels 38MnSiV5 and 16MND5.[9] The first alloy shows a very flat temperature variation which the model cannot explain, but there is a very good agreement for the second alloy.

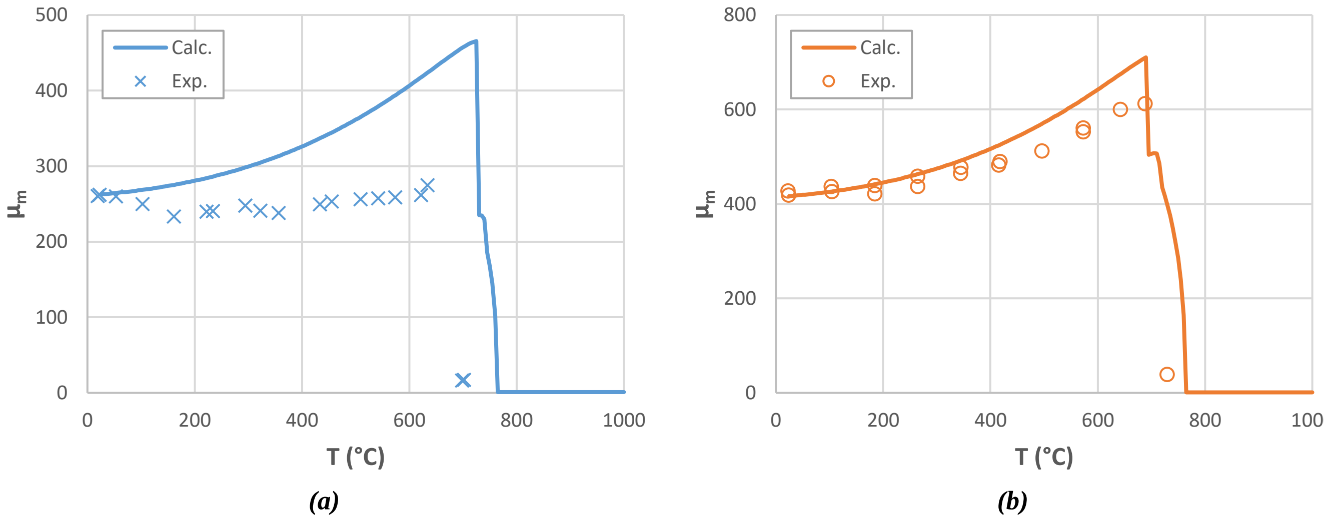


Figure 2: *Maximum permeability as a function of temperature for steels 38MnSiV5 (a) and 16MND5 (b). The solid curves are calculated using JMatPro® while the symbols are experimental results from Ref. 9.*

Permeability as a Function of Magnetic Field Strength

In addition to thermal gradients, nonuniform distributions of magnetic field strength are formed across the width of the workpiece undergoing induction heating.[1] For ferromagnetic steels this leads to complex changes in the magnetic permeability, which are important to quantify.

The first step in the calculation of magnetic permeability as a function of applied field strength is to invoke the linear mixing model of Eq. (1) to decouple the contribution of ferrite transformation to other phases.

Based on the works of Mel'gui[10] and Sandomirskii,[11] an accurate description of the magnetic field-dependent permeability for the ferritic component of the microstructure is then written as:

$$\mu_f(H) = 1 + \frac{4\pi\chi_i^f}{h^2+1} + \frac{B_s}{H_c} \cdot \frac{h}{\pi(h^2+b)} \cdot \left[\tan^{-1}\big(a(h-1)\big) + \tan^{-1}\big(a(h+1)\big)\right] \tag{6}$$

In the above expression, $h = H/H_c$, the parameters a and b are used to match the value and position of the maximum permeability [Eq. (2)], and χ_i^f is the initial magnetic susceptibility for ferrite, which is related to the initial permeability via[2]

$$\mu_i^f = 1 + 4\pi\chi_i^f \tag{7}$$

As for the maximum permeability, it is convenient to relate μ_i^f to other more readily available magnetic properties. In this case, a simple relationship is obtained in terms of H_c and the residual magnetic flux density or remanence, B_r, rather than B_s:

$$\mu_i^f = 1 + \frac{B_r}{8H_c} \tag{8}$$

Figure 3 illustrates the validity of Eq. (8) using room-temperature data for low-alloy steels.[12,13] It is also possible to establish a relationship between B_r, B_s, and H_c from an empirical formula given by Sandomirskii:[11]

$$B_r \approx \frac{B_s}{2 + \frac{0.06H_c}{\pi} - \left(\frac{0.068H_c}{2\pi}\right)^2} \tag{9}$$

For typical values of room-temperature hardness, B_r is about 2–2.5 times lower than B_s, which agrees with experimental observations for soft magnetic steels.[4,12] As the hardness (and, thus, the coercivity) increases, the term in $-H_c^2$ allows for the ratio to start inverting, giving values closer to B_s as expected for hard magnets.[2,11]

Combining Eqs. (1) and (6) through (9), gives the final result:

$$\mu(H) = 1 + v_f \cdot \frac{B_s}{H_c} \cdot \left[\frac{1}{c(h^2+1)} + \frac{h}{\pi(h^2+b)} \cdot \left[\tan^{-1}\big(a(h-1)\big) + \tan^{-1}\big(a(h+1)\big)\right]\right] \tag{10}$$

where

$$c = 16 + \frac{0.48H_c}{\pi} - 2\left(\frac{0.068H_c}{\pi}\right)^2 \tag{11}$$

While Eq. (10) has been derived from empirical relationships established at room temperature, it is assumed that the same form holds at elevated temperatures. The effect of temperature is included explicitly via the terms $v_f(T)$, $B_s(T)$, and $H_c(T)$, as discussed in the previous section. Note that the parameters a, b, and c also contribute to the temperature variation through their dependence on μ_m^f and H_c.

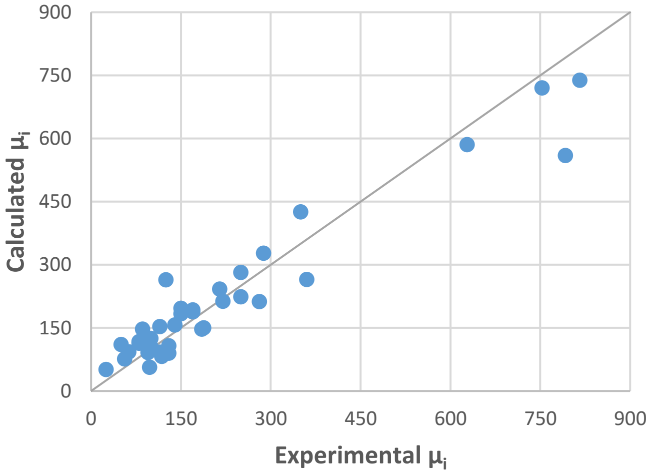
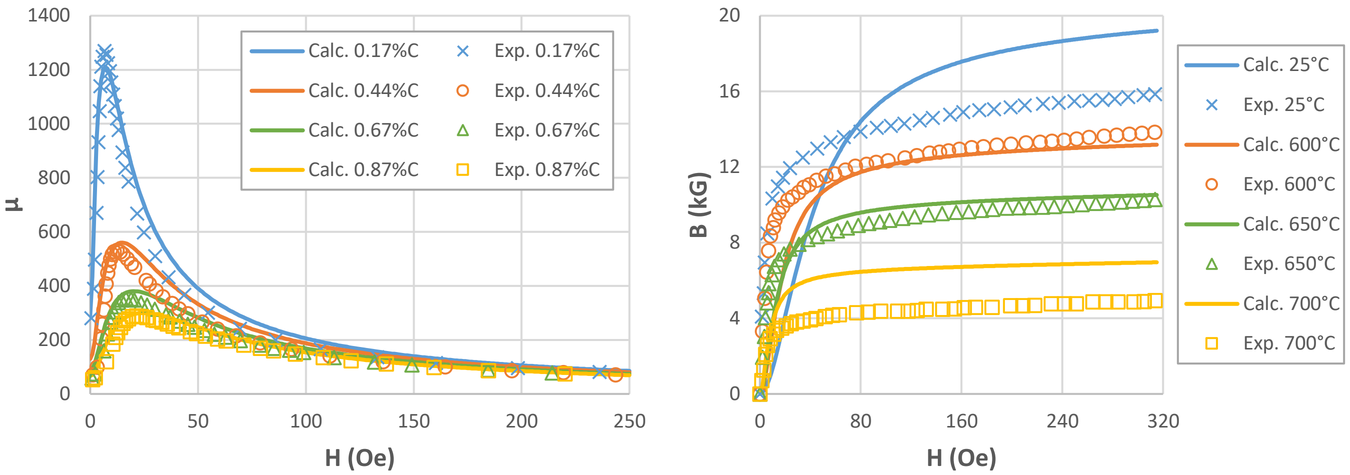


Figure 3: *Comparison of predictions for initial permeability with room-temperature experimental measurements. The data used are from Refs. 12 and 13.*



(a) ***(b)***

Figure 4: *Permeability (a) and flux density (b) as a function of field strength for carbon-sulphur steels. The solid curves are calculated using JMatPro® while the symbols are experimental results from Refs. 13 and 14.*

The model predictions for the permeability and flux density, $B = \mu \cdot H$, as a function of applied field strength are compared with experimental results for several carbon-sulphur steels[13,14] in Fig. 4. As can be seen in Fig. 4(a), the permeability curves obtained for varying carbon content at room temperature are in excellent agreement with experiments. For the calculated magnetisation curves of Fig. 4(b), the saturation regimes are pushed to higher field strength values and, also, the predicted saturation values can be somewhat higher than the observed ones, especially at room temperature. This prevents a perfect match with experiments, but there is reasonable agreement which justifies the assumptions used to include the effect of temperature in the model.

Summary

The model used in JMatPro® to calculate the magnetic permeability of general steels has been described. It includes the main factors that affect the temperature and magnetic field dependencies of the permeability: Curie temperature, coercivity and its mechanisms as effectively encompassed in the mechanical hardness, and the effect of phase transformations and changes in the heating rate.

The method requires only a limited number of user inputs, which include the heating rate and the alloy's composition, microstructure, and room-temperature hardness. Accurate results are expected for low-alloy steels at room temperature, but, in general, at least useful trends should be provided. Such calculations should facilitate the selection of optimal frequencies for induction heating applications, and mitigate the lack of material data required for process simulations, particularly if complex temperature and magnetic field strength distributions are considered.

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